

Math 3751 Problems
Spring 2016

1. **(Due 1/27/16)** Suppose that A and B are bounded nonempty sets. Show that $\sup(A + B) = \sup A + \sup B$, where

$$A + B = \{z : z = x + y \text{ for some } x \in A \text{ and } y \in B\}.$$

2. **(Due 2/3/16)** Using the definition of the limit of a sequence, show that

$$\lim \frac{n^2 - 4n + 2}{5n^2 - 3} = \frac{1}{5}.$$

3. **(Due 2/8/16)** Suppose $L = \lim a_n$ and let $s_n = (a_1 + a_2 + \dots + a_n)/n$. Show that $\lim s_n = L$.

4. **(Due 2/12/16)** (a) Let $x_1 = 2$ and $x_{n+1} = \sqrt{\frac{x_n+1}{2}}$ for each $n \in \mathbf{N}$. Show that $\{x_n\}$ converges.

(b) Let $y_1 = 1$ and $y_{n+1} = y_n x_n$ for each $n \in \mathbf{N}$. Show that $\{y_n\}$ is increasing. (For those enrolled in MATH 5851: Show that $\{y_n\}$ converges.)

5. **(Due 2/26/16)** Do problems 4.2.10 and 4.2.11 from the text (Thomson & Bruckner).

6. **(Due 3/4/16)** (a) Suppose that E is closed and define $U_n = \cup_{x \in E} (x - \frac{1}{n}, x + \frac{1}{n})$ for each $n \in \mathbf{N}$. Show that $E = \cap_{n=1}^{\infty} U_n$.

(b) Show that if U is an open set, then $U = \cup_{n=1}^{\infty} F_n$ for some sequence $\{F_n\}$ of closed sets. (Think complements.)

7. **(Due 3/18/16)** Using the definition of limit, show that

$$\lim_{x \rightarrow 3} \frac{x + 5}{4x - 11} = 8.$$

8. **(Due 3/25/16)** Show that if f is continuous on each closed set X_i for $i = 1, 2, \dots, n$, then f is continuous on $X = \cup_{i=1}^n X_i$. For graduate students: Show that this is not necessarily true if the number of sets X_i is infinite.

9. **(Due 4/11/16)** Show that $f(x) = \sqrt{x}$ is uniformly continuous on $[0, \infty)$.

10. **(Due 4/15/16)** Suppose that f is increasing and continuous on $[a, b]$. Show that f^{-1} is increasing and continuous on $[f(a), f(b)]$.

11. **(Due)** (a) Suppose that f' is increasing on \mathbf{R} . Show that the graph of f lies above any of its tangent lines.

(b) Do problem 7.7.5 from the text (Thomson & Bruckner).