

Using the method of Laplace transforms solve the following initial value problems.

1.  $y' - y = 3te^{-2t}; \quad y(0) = -2$

2.  $y'' + 4y' + 3y = u(t - 1) - u(t - 3); \quad y(0) = 1, \quad y'(0) = 2$

3.  $y'' + 4y = 3 \sin t; \quad y(0) = 3, \quad y'(0) = -4$

4. Find the Laplace transform of the following function.

$$f(t) = \begin{cases} t, & \text{if } 0 < t < 4, \\ 4e^{-t+4}, & \text{if } t > 4. \end{cases}$$

Bonus. Find the inverse Laplace transform of  $F(s) = \ln \left( \frac{s^2 - 1}{s^2 + 4} \right)$ . Hint: See problems 33–36 of section 7.4.