

Ninth Annual Calculus Competition

May 9, 1998

1. Find all values for a such that $y = 3x + 1$ is tangent to $y = ax^2 + 4x$.
2. Consider all line segments that pass through the point $(3, 2)$ whose endpoints lie on the positive coordinate axes. Find the slope of the shortest such line segment.
3. Show that $x^3 + 2x + k$ has exactly one zero for all real numbers k .
4. Evaluate: $\lim_{x \rightarrow 0} \frac{\int_0^x \sin(t^2) dt}{x(1 - \cos x)}$
5. Suppose f , f' , and f'' are continuous on $[0, 1]$ and that $f'(e) = f(e) = f(1) = 1$ and $\int_1^e \frac{f(x)}{x^2} dx = \frac{1}{2}$. Find the value of $\int_1^e f''(x) \ln x dx$.
6. Find the volume of a pyramid with center height 3 and a square base with edge of length 4.
7. A cup of coffee in a 70°F room has an initial temperature of 190°F and in 5 minutes cools to 160°F . If the coffee cools at a rate proportional to the temperature difference between the coffee and the room, how long does it take the coffee to cool to 110°F .
8. Let $f(x) = \sum_{n=0}^{\infty} \frac{4}{(x^2 + 2)^n}$. Determine all x for which $f(x) > 5$.
9. A box made of wood is to have an inside volume of 4ft^3 . If the wood is $1/2$ inch thick on the top and bottom and 1 inch thick on the four sides, find the dimensions of the box that uses the least amount of material (by volume).
10. Evaluate: $\int_0^2 \int_{y^2}^4 y \cos(x^2) dx dy$