Eighth Annual Calculus Competition

May 3, 1997

1. Sketch the graph of $y = e^{\ln[(x+3)/(x+1)]}$

2. Find all constants a and b such that $y = 2x - 3$ is tangent to the graph of $f(x) = ax^2 + bx$.

3. Evaluate: $\displaystyle \lim_{x\to\infty} \frac{\int_0^x \sqrt{t^4+3t^2}\, dt}{x^3+3x}$

4. Consider the collection of all curves of the form $y = a - bx^2$ that pass through the point $(2,1)$, where a and b are positive constants. Determine the values of a and b that will minimize the area of the region bounded by $y = a - bx^2$ and the x-axis.

5. A leaky bucket is used to draw water from a well that is 60 ft deep. The bucket weighs 2 lb and the rope attached to it weighs 0.5 lb/ft. The bucket starts with 40 lb of water and reaches the top of the well with 34 lb. Assuming that both the rate at which the water is leaking and the rate at which the bucket is drawn upward are constant, find the work done in pulling the bucket to the top of the well.

6. Determine the sum of $\displaystyle \sum_{n=2}^{\infty} \left[(n+1)\sin\left(\frac{\pi}{n+1}\right) - n\sin\left(\frac{\pi}{n}\right) \right]$.

7. A ball is dropped from a height of 5 ft. Assuming that on each bounce the ball returns to 60% of its height attained on the previous bounce and that the ball continues to bounce indefinitely, find the total distance the ball will travel.

8. A tank contains 500 gallons of brine with 20 lb of dissolved salt. Pure water enters the tank at the rate of 2 gal/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt is left in the tank after 10 minutes.

9. Find the point(s) on the surface $z = x^2 + 4y^2 - 4$ that are the closest to the origin.

10. Evaluate: $\displaystyle \int_0^4 \int_{-\sqrt{4-x}}^{\sqrt{4-x}} \frac{1}{\sqrt{20+12y-y^3}}\, dydx$