Seventh Annual Calculus Competition

May 4, 1996

1. Find all points on the graph of $y = x^2$ such that the lines tangent to the curve at those points pass through $(0, -2)$.

2. Evaluate: $\lim_{n\to\infty} \sqrt{n^2 + 4n} - n$.

3. Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where $a > 0$ and $b > 0$.

4. Suppose f, f', and f'' are continuous on $[0, 3]$ and that $f(3) = 1$, $f'(3) = 2$, and $\int_0^3 f(x)dx = 7$. Find the value of $\int_0^3 x^2 f''(x)\, dx$.

5. Evaluate the improper integral $\displaystyle\int_0^\infty \frac{1}{(x-2)^3}\, dx$, or show that it diverges.

6. Find the values for a such that $\ln(x) = ax$ has exactly one solution.

7. Evaluate: $\displaystyle\lim_{x\to\infty} \frac{\int_0^x t\sqrt{9 + t^4}\, dt}{x^4}$

8. Find all values of x for which the series $\displaystyle\sum_{n=0}^{\infty} \left(\frac{2x}{1+x}\right)^n$ converges.

9. Find a vector tangent to the curve of intersection of the surfaces $x + y = 2$ and $z = x^2 + y^2$ at $(1, 1, 2)$.

10. Evaluate: $\displaystyle\int_0^1 \int_{2y}^2 e^{-x^2}\, dx dy$.