Sixth Annual Calculus Competition

April 29, 1995

1. Suppose that f is differentiable and $g(x) = x^2 f(\frac{1}{x})$. If $f(1) = 3$ and $f'(1) = -5$, what is $g'(1)$?

2. Find all asymptotes of the graph of $y = \dfrac{\sqrt{x^4+1}}{x-1}$.

3. Determine a cubic polynomial that passes through $(0,0)$ and $(2,1)$ and has a relative maximum at $(2,1)$ and a point of inflection at $(0,0)$.

4. Evaluate: $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{n}\sqrt{1+\frac{3k}{n}}$.

5. Evaluate: $\displaystyle\lim_{x\to 0^+} (\cos x)^{\frac{1}{x^2}}$.

6. Evaluate: $\displaystyle\int_{-4}^{-2} \frac{\sqrt{x^2-4}}{x}\, dx$.

7. Determine the sum of $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$.

8. Show that for $x \geq 0$, $x - \dfrac{x^2}{2} \leq \ln(1+x) \leq x$.

9. A rectangular box with a volume of 6 cubic feet is to be constructed from three different materials. If the top and bottom cost \$6 per sq. ft., the front and back cost \$4 per sq. ft., and the sides cost \$2 per sq. ft., what are the dimensions of the cheapest such box that can be made?

10. Evaluate: $\displaystyle\int_1^3 \int_{\ln y}^{\ln 3} \frac{x}{e^x - 1}\, dx dy$.