

Fourth Annual Calculus Competition

February 13, 1993

1. Determine all asymptotes to the graph of the equation $xy + 3|x| - y = 0$.
2. Evaluate the improper integral $\int_0^{\infty} \frac{dx}{(x-1)^2}$ if it converges.
3. Evaluate: $\lim_{x \rightarrow 1} \frac{\int_1^x \sqrt{1+4t^2} dt}{x-1}$
4. Let $f(x) = \frac{3x^4 + 16}{2x^4 + 11}$. Find the whole number nearest to $f(2^{32})$.
5. For which values of a does the line $y = ax$ intersect the graph of $y = e^x$ exactly once.
6. Consider the region in the first quadrant bounded by $y = x^n$ (where $n > 1$), the x -axis, and the tangent line to the graph of $y = x^n$ at the point $(1, 1)$. Find the value of n for which the enclosed area is a maximum. (Note: n is not necessarily an integer.)
7. Suppose f, f' , and f'' are continuous on $[0, \pi]$ and that $f(0) = 1, f(\pi) = 3$, and $\int_0^{\pi} f''(x) \sin x dx = \pi$. Find the value of $\int_0^{\pi} f(x) \sin x dx$.
8. Find all values of x for which the series $\sum_{k=0}^{\infty} \frac{2^k x^k}{\ln(k+2)}$ converges.
9. Find a vector tangent to the curve of intersection of the surfaces $x^2 - y^2 - 2z^2 = 1$ and $3x + y + 6z = 1$ at $(2, 1, -1)$.
10. Evaluate $\int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\cos y}{y} dy dx$.