Second Annual Calculus Competition

February 9, 1991

1. Find $f'(\sqrt[4]{\pi})$, if $f(x) = \int_0^{x^2} \sin(t^2)\ dt$.

2. Evaluate: $\int_0^1 \int_0^{\arccos y} \frac{1}{\sqrt{4+\sin x}}\ dxdy$

3. Suppose that the second derivative of f exists everywhere and that $f(x_1) = f(x_2) = f(x_3) = 0$, where $x_1 < x_2 < x_3$. Show that $f''(c) = 0$ for some number c with $x_1 < c < x_3$.

4. Suppose that $f(x) = x^3 + x$ and that g is the inverse function for f. Find $g'(2)$.

5. Determine a cubic polynomial that passes through $(0,0)$ and $(1,1)$ and has a relative minimum at $(0,0)$ and a point of inflection at $(1,1)$.

6. Determine an equation of the plane that is tangent to the surface with equation $x^2 + y^2 - 3z = 2$ at the point $(-2,-4,6)$.

7. Let $f(x) = e^{-1/x}$, where $x > 0$. Find $\lim_{x \to 0^+} f'(x)$.

8. Let $A(1,0,1)$, $B(-1,2,3)$, and $C(4,4,1)$ be the vertices of a triangle. Find the length h of the altitude of the triangle that has A as an endpoint.

9. Write an expression for the nth derivative, $f^{(n)}(x)$, if $f(x) = \frac{1}{x+2}$.

10. Determine the value of the sum $\sum_{n=2}^{\infty} \frac{2^{n+1}}{3^{n-1}}$.