

Thirtieth Annual Calculus Competition

April 11, 2019

1. Let f be a function such that $f'(1) = -1$ and $f''(1) = 3$, and let $g(x) = f(\tan x)$. Find $g''(\pi/4)$.
2. Find a polynomial $P(x)$ of degree 3 such that $P(2) = P''(2) = 0$ and $P'(2) = P'''(2) = 6$.
3. Suppose that a and b are positive numbers such that the point (a, b) lies on the curve $x^2y = 6$. Find the smallest possible value of $8a^2 + 3b^2$.
4. Evaluate: $\int_1^{100} \lfloor x \rfloor \cdot (-1)^{\lfloor x \rfloor} dx$, where $\lfloor x \rfloor$ denotes the least integer not less than x .
5. Suppose that $f(x) = x^3 - 3x^2 + 12x$ and that g is the inverse function for f . Find $g'(10)$.
6. Evaluate: $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{\sqrt{4n^2 - k^2}}$.
7. Find a value for x such that $\sum_{n=1}^{\infty} nx^n = 1$.
8. Find a point P on the surface $z = x^2 - 4y^2$ such that the plane tangent to this surface at P is parallel to the plane $6x - 8y - z = 1$.
9. Let D be the region $\{(x, y) : 4 \leq x^2 + y^2 \leq 9 \text{ and } y \geq 0\}$. Evaluate: $\iint_D e^{x^2+y^2} dA$.
10. Evaluate: $\int_0^4 \int_{\sqrt{x}}^2 \frac{6xy}{\sqrt{36 + y^6}} dy dx$.