Twenty-Ninth Annual Calculus Competition

April 5, 2018

1. Find all points P on the curve $y = 1 + x^2$ for which the line tangent to this curve at P passes through the point $(1, 0)$.

2. Show that $x^4 \geq 32x - 48$ for all real numbers x.

3. Consider all line segments that pass through the point $(8, 5)$ whose endpoints lie on the positive coordinate axes. Find the slope of the shortest such line segment.

4. Let $\mathcal{R}$ be the region bounded by $y = 4x - x^2$ and the x-axis. Find m such that the line $y = mx$ divides $\mathcal{R}$ into two regions with equal areas.

5. Suppose that $\int_0^1 f(x)\, dx = 4 = f'(1)$ and $f(1) = 3$. Find the value of $\int_0^1 x^2 f''(x)\, dx$.

6. Evaluate: $\displaystyle\lim_{x \to 0} \frac{\int_0^x \sin(t^2)\, dt}{x - \sin x}$

7. Evaluate: $\displaystyle\int_0^\infty \frac{1}{e^x + a}\, dx$, where a is a constant.

8. Find the largest integer that is less than or equal to $\dfrac{6!}{e}$.

9. Let P be a point on the line $\frac{x-4}{2} = \frac{y-1}{3} = \frac{z-2}{2}$ and Q be a point on the sphere $x^2 + y^2 + z^2 - 4x + 6z = -4$. Find the minimum distance between P and Q.

10. Evaluate: $\displaystyle\int_0^4 \int_{\sqrt{y}}^2 3\sqrt{1 + x^3}\, dx\, dy$.