

Twenty-Eighth Annual Calculus Competition

April 6, 2017

1. Find all values A such that the polynomial $x^3 - 3x^2 - 9x + A$ has three distinct real zeros.
2. Let $f(x) = \frac{7x^3 + 10}{2x^3 + 3}$. Find the whole number nearest to $f(5^{16})$.
3. Let P be a point in the first quadrant on the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$, and let l be the line tangent to this curve at this point. Find the smallest possible area of the triangle whose sides lie on l , the x -axis, and the y -axis.
4. Suppose that $f'(x) = \frac{1}{\sqrt{x^3 + 1}}$ for all $x > -1$ and that $f(2) = 1$. Find the value of $\int_0^2 xf(x) dx$.
5. Evaluate: $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{4\sqrt{n^2 - k^2}}{n^2}$
6. Find a real number k that minimizes the integral $\int_0^4 (x^2 - k)^2 dx$.
7. Evaluate: $\lim_{x \rightarrow \infty} \left(\int_1^x \frac{e^t}{t^2} dt \right)^{1/x}$
8. Let $f(x) = \arctan x$. Find the value of $f^{(2017)}(0)$.
9. Let P and Q be points on the surfaces $x^2 + y^2 + z^2 - 4x = 5$ and $2x - 2y + z = 19$, respectively. Find the minimum distance between P and Q .
10. Evaluate: $\int_0^1 \int_0^{\arccos y} \sqrt{1 + 3 \sin x} dx dy$.