

Twenty-Seventh Annual Calculus Competition

April 7, 2016

1. Find a cubic polynomial $P(x)$ satisfying the following: $P(0) = 3$; $P(1) = 2$; and $P'(0) = P'(1) = 0$.
2. Evaluate:
$$\lim_{x \rightarrow 1} \frac{\int_x^{x^2} e^{\sqrt{t}} dt}{x - 1}$$
3. Evaluate:
$$\int_0^1 \arcsin(x) dx$$
4. Let a be a real number larger than 1. Find the minimum value of $f(x) = xa^{1/x}$ on the interval $(0, \infty)$.
5. For what value of c does the line $x = c$ divide the region under the curve $y = e^x$ between $x = 0$ and $x = 3$ into equal areas.
6. Show that the equation $3x^5 - 10x^3 + 15x - 4 = 0$ has exactly one real solution.
7. Find all points P on the curve, given parametrically by $x = t^2 - 2t$ and $y = t^2 + t + 1$, for which the line tangent to the curve at P passes through the point $(2, 0)$.
8. Find all values of x for which the series $\sum_{n=1}^{\infty} \frac{1}{n^2} \left(\frac{x^2 - 5}{4} \right)^n$ converges.
9. Let P and Q be points on the lines $\frac{x-4}{2} = \frac{y-1}{3} = z + 1$ and $\frac{x-2}{3} = \frac{y}{4} = \frac{z+2}{2}$, respectively. Find the minimum distance between P and Q .
10. Evaluate:
$$\int_0^2 \int_x^2 x \sqrt{9 - y^3} dy dx.$$