

Twenty-Fifth Annual Calculus Competition

April 10, 2014

1. The line tangent to $y = x^4 - 8x^3 + 10x^2$ at $(2, -8)$ cuts this curve in two points. Find the x -coordinates of these points.

2. Evaluate:
$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{n}{k^2 + n^2}$$

3. Let n be a positive integer. Find the maximum value of the function $f(x) = \left(\frac{n}{x}\right)^x$ on the interval $(0, \infty)$.

4. Suppose that f' is continuous, $f(1) = 2$, and $f'(1) = 3$. Evaluate:

$$\lim_{x \rightarrow 2} \frac{f(7x - 13) - f(3x - 5)}{x^2 - 4}.$$

5. Find the function f such that $f(0) = 3$ and for each real number c the line tangent to $y = f(x)$ at $(c, f(c))$ has $c - 2$ as its x -intercept.

6. Find the volume of the solid of revolution obtained by revolving about the x -axis the region bounded by the x -axis, the line $x = e$, and the curve $y = \ln(x)$.

7. The geometric series $\sum_{n=0}^{\infty} ar^n$ has a sum of 23, and the terms involving odd powers of r have a sum of 9. Find the value of a .

8. Let $J(t) = \int_0^{\pi} \ln(1 - 2t \cos x + t^2) dx$. Show that $J(t)$ is an even function of t .

9. Let P and Q be points on the surfaces $x - 2y + 2z = 12$ and $x^2 + y^2 + z^2 - 2x - 4y = 4$, respectively. Find the minimum distance between P and Q .

10. Evaluate:
$$\int_0^4 \int_0^{\sqrt{y}} \frac{\sqrt{y}}{6 + 32x - x^4} dx dy.$$