Twenty-Second Annual Calculus Competition

April 16, 2011

1. Find the third degree polynomial whose graph is tangent to the line $y = 1$ at $(0, 1)$ and tangent to the line $y = 12x - 19$ at $(2, 5)$.

2. Find all values of m for which the equation $x^3 + 2x^2 - 4x = m$ has three distinct real solutions.

3. Consider all line segments that pass through the point $(4, 3)$ and whose endpoints lie on the positive coordinate axes. Find the slope of the line segment for which the sum of its x and y intercepts is a minimum.

4. Suppose that f is continuous and satisfies $f(x) + f(4 - x) = (\cos \pi x)^2$ for each x. Find $\int_0^4 f(x) \, dx$.

5. Evaluate: $\displaystyle \lim_{x \to 0} \frac{\int_0^x \sqrt{t^3 + 1} \, dt - x}{x^4}$.

6. Find the function f defined on $(0, \infty)$ that satisfies $f(x) = 1 + \frac{1}{x} \int_1^x f(t) \, dt$.

7. A drinking cup with height 16 cm has a circular base with radius 3 cm and a top with radius 5 cm. If the cup is half full, what is the depth of the liquid within it?

8. Evaluate: $\displaystyle \lim_{x \to 0} \left(\frac{e^x + e^{-x}}{2} \right)^{1/x^2}$.

9. Determine a value for C so that $\displaystyle \int_3^\infty \frac{Cx}{x^2 + 1} - \frac{1}{3x + 1} \, dx$ converges. Then evaluate the integral for this value of C.

10. Evaluate: $\displaystyle \int_0^1 \int_{\sqrt{y}}^1 \frac{x^7}{\sqrt{1 + x^3 y}} \, dx \, dy$.