

## Twentieth Annual Calculus Competition

April 11, 2009

1. Determine a cubic polynomial whose graph is tangent to  $y = 2x + 1$  at  $(0, 1)$  and to  $y = 6 - 2x$  at  $(3, 0)$ .
2. Find the radius of the largest circle that can be inscribed in the region bounded by  $y = 4 - x^2$  and the  $x$ -axis.
3. Find the volume of the solid generated by rotating about the  $y$ -axis the region bounded by  $y = -2x^2 + 9x - 7$  and  $y = -x^2 + 5x - 4$ .

4. Let  $f(x) = \int_0^{4-x^2} \frac{t^2 - 2t - 3}{\sqrt{t^2 + 4}} dt$ . Find the intervals on which  $f$  is increasing.

5. Evaluate:  $\int \frac{\sqrt{x^2 + 25}}{x} dx$ .

6. Evaluate:  $\lim_{x \rightarrow 0^+} (2e^x - e^{2x})^{1/x^2}$

7. Find (or estimate) the smallest integer  $N$  such that  $\sum_{k=1}^N \frac{1}{\sqrt{k}} \geq 10$ .

8. Determine the values of  $x$  for which the series  $\sum_{n=1}^{\infty} \frac{1}{n} \left( \frac{1-x}{1+x} \right)^n$  converges.

9. Find the largest value of  $f(x, y, z) = 9x + 16y + 25z$ , where  $x$ ,  $y$ , and  $z$  are positive real numbers satisfying  $x^3 + y^3 + z^3 = 1$ .

10. Evaluate:  $\int_{-2}^2 \int_{|x|}^2 x^2 \sqrt{16 - y^4} dy dx$ .