

Nineteenth Annual Calculus Competition

April 5, 2008

1. Suppose that $f(0) = 0$ and $f'(0) = 2$, and let $g(x) = f(-x + f(f(x)))$. Find $g'(0)$.
2. Find all values of b for which the line tangent to the graph of $f(x) = x^3 - \frac{11}{3}x$ at the point $(b, f(b))$ passes through the point $(3, 0)$.
3. Consider all triangles POQ , where O is the origin, P lies on the positive x -axis, Q lies on the positive y -axis, and the segment \overline{PQ} contains the point $(3, 1)$. Find the smallest area that such a triangle can have.
4. Evaluate:
$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{18n}{(n+2k)^2}$$
5. Find a function f such that $f(0) = 2$ and for each real number a the line tangent to $y = f(x)$ at $(a, f(a))$ has x -intercept $a - 2$.
6. Evaluate:
$$\int_0^{12} \frac{x^2}{x^2 + (12-x)^2} dx$$
7. Evaluate:
$$\lim_{x \rightarrow 0} \frac{\ln(\sec 6x)}{3x^2}$$
8. Find the length of the curve $x = 2 \cos t + \cos 2t$, $y = 2 \sin t - \sin 2t$, $0 \leq t \leq 2\pi$.
9. Find the sum of
$$\sum_{n=0}^{\infty} \frac{2n+3}{3^n}$$
.
10. Let $f(x) = \int_x^2 \frac{1}{\sqrt{1+y^3}} dy$. Evaluate: $\int_0^2 x f(x) dx$.