Twelfth Annual Calculus Competition

April 14, 2001

1. Suppose that f is a differentiable function such that $f(2) = 1$ and $f'(2) = 3$ and let $g(x) = f(xf(x))$. Find $g'(2)$.

2. Let $A(x_0)$ be the area of the triangle whose sides are the coordinate axes and the line tangent to $8x^2 + 50y^2 = 400$ at the point (x_0, y_0), where (x_0, y_0) is in the first quadrant. Find the minimum value of $A(x)$.

3. Evaluate: $\displaystyle\lim_{n\to\infty} \sum_{k=1}^{n} \frac{4}{n+4k}$

4. Find the area of the region bounded by the curves $y = x^3 - x^2 - x + 1$ and $y = x + 1$.

5. Suppose that f, f', and f'' are continuous on $[0, \ln 2]$ and that $f(0) = 0$, $f'(0) = 3$, $f(\ln 2) = 6$, $f'(\ln 2) = 4$, and $\int_0^{\ln 2} e^{-2x} f(x)\, dx = 3$. Find the value of $\int_0^{\ln 2} e^{-2x} f''(x)\, dx$.

6. Find the arc length of the curve $y = \ln x$, where $1 \le x \le \sqrt{3}$.

7. Find all lines that pass through the point $(1, 1)$ and are tangent to the curve $x = 2t - t^2$, $y = t + t^2$.

8. Find two numbers A and B such that $B - A < 0.001$ and $A < \displaystyle\sum_{n=11}^{\infty} \frac{1}{n^3} < B$.

9. Find a value for r such that the surface $x^2 + y^2 + z^2 = r^2$ is tangent to the surface $z = x^2 + 3y^2 - 2$ at a point not on the z-axis.

10. Evaluate: $\displaystyle\int_0^1 \int_y^1 \frac{xy}{\sqrt{1+x^4}}\, dx dy$