Eleventh Annual Calculus Competition

May 6, 2000

1. Find all points (x_0, y_0) on the curve $y = x^2 + \frac{3}{4}$ such that the line tangent to this curve at (x_0, y_0) passes through $(\frac{1}{2}, 0)$.

2. Find the area of the region that lies inside both $r = 1 + \cos\theta$ and $r = 1 - \cos\theta$.

3. Show that $x^2 \leq 4e^{x-2}$ for all $x \geq 0$.

4. Evaluate: $$\lim_{x\to 0} \frac{\int_0^x \sin t^2 \, dt}{2\sin(x) - \sin(2x)}$$

5. Evaluate: $$\int \frac{1}{\ln(x^x e^x)} \, dx$$

6. Find a real number k that minimizes the integral $\displaystyle\int_0^1 |x^2 - k^2| \, dx$.

7. Find the volume of the solid generated by rotating about the y-axis the region inside the circle $(x-4)^2 + y^2 = 4$ and outside the circle $(x-4)^2 + y^2 = 1$.

8. Let $f(x) = \displaystyle\sum_{n=0}^{\infty} \left(\frac{3x-1}{7}\right)^n$. Determine all x such that $f(x) \geq 1$.

9. Evaluate: $$\int_0^2 \int_y^2 y\sqrt{1+x^3} \, dx dy$$

10. Find the point at which the plane tangent to $x^2 - 3xy + 4z^2 = 2$ at $(2, 1, 1)$ intersects the z-axis.