

Tenth Annual Calculus Competition

May 8, 1999

1. Suppose that $f(1) = 3$, $f'(1) = -1$, $h(3) = 2$, and $h'(3) = 5$. If $g(x) = h(x^2f(x))$, find $g'(1)$.
2. Find all points of intersection of the line tangent to $f(x) = x^4 + 6x^3 + x - 1$ at $x = 1$ with the curve $y = f(x)$.
3. Find the point in the first quadrant on the graph of $y = \frac{4}{x^2}$ that is closest to the origin.

4. Evaluate: $\lim_{x \rightarrow \infty} \frac{\int_0^x \sqrt{4+t^4} dt}{x^3}$

5. Evaluate: $\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{2}{n} \left(\frac{2k}{n} + 1 \right)^3$

6. Find all values of x such that $\sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{x-5}{3} \right)^n$ converges.

7. A cylindrical hole with radius 3'' is drilled through a sphere with radius 5'' so that the central axis of the hole lies on a diameter of the sphere. Find the volume of the material remaining.
8. The graph of a certain function f contains the point $(0, 2)$ and has the property that for each number p the line tangent to $y = f(x)$ at $(p, f(p))$ intersects the x -axis at $p + 2$. Find $f(x)$.
9. Find a point $P \neq (0, 0, 0)$ for which the plane tangent to $z = x^2 + y^2$ at P intersects the xy -plane in the line $x - 2y = 3$.

10. Evaluate: $\int_0^4 \int_0^{\sqrt{4-y}} \sqrt{12x - x^3} dx dy$